A Benchmark for Control of Anti-lock Braking Systems

Stefan Solyom[1], Anders Rantzer[1] and Jens Kalkkuhl[2]

[1]Lund Institute of Technology, Dept. of Automatic Control, Lund, Sweden

[2]DaimlerChrysler Research and Technology, Stuttgart, Germany

The following is a simplified model of wheel dynamics subject to brake torque and ground contact forces.

$$\begin{aligned}\frac{d\omega(t)}{dt} &= \alpha\mu\left(\lambda(t)\right) - \beta T_b(t-\tau)\mu_b(\omega(t)), \quad T_b \geq 0\\ \frac{dv(t)}{dt} &= -\gamma\mu(\lambda(t))\\ \lambda(t) &= \frac{v(t)-\omega(t)r}{v(t)}\end{aligned}$$

where:

ω - angular velocity of the wheel

v - velocity over ground of the car

λ - longitudinal tire slip

T_b - brake torque. It is the input signal of the model.

μ - road-tire friction coefficient. Dependence on the longitudinal tire slip (λ) for four different surfaces is shown in Figure 1.

μ_b - friction coefficient in the brakes. This is used in the modeling of wheel lock. For simplicity use $\mu_b = \min(\omega/\varepsilon, 1)$, for some small $\varepsilon > 0$.

τ - time delay (an appropriate value is 14 miliseconds)

r - wheel radius (a suitable value for a passenger vehicle is 0.3 meters)

α, β, γ - positive constants, resulting from physical parameters of the vehicle (appropriate values for a passenger vehicle are respectively 1500, 1 and 10).

The following signals are available from the plant:

- ω - angular velocity of the wheel
- v - velocity over ground of the car
- μ_H - maximum road-tire friction coefficient

These are estimated or measured and available for feed-back in the control algorithm. In this benchmark, all of these signals can be considered measurable.

Specifications

The control objective is to maintain a desired tire slip level λ, by adjusting the brake torque (T_b). The ABS has to fulfill the following requirements [8]:

- no wheel lock allowed to occur for speeds above $4\frac{m}{s}$
- wheel lock for a period of less than 0.2 seconds is allowed for speeds in the range of $0.8 \ldots 4\frac{m}{s}$
- the control system should be robust with respect to other unmodeled dynamics, e.g. actuator dynamics. A reasonable model of the actuator dynamics, that can be used in the robust design is:

$$\frac{.0091s + 3.9545}{0.0001s^2 + 0.0402s + 3.9545} .$$

It is of special importance that the above mentioned specifications are fulfiled in case of transition between different surface conditions while braking (e.g. transition from dry to wet surface).

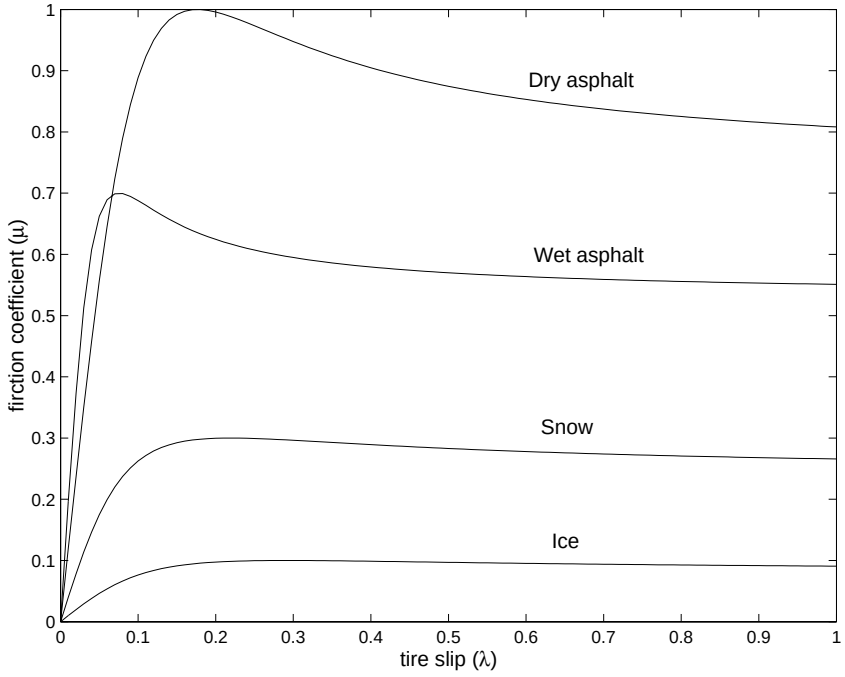

Figure 1 Tire friction curves

Comments on the model

The benchmark model is derived from the well known quarter car model [6], [4]. This consists of a single wheel attached to a mass, as shown in Figure 2.

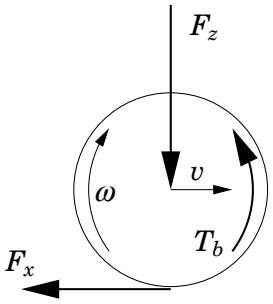

Figure 2 Quarter car

The benchmark describes the equations of motion of the quarter car in case of braking. The constants given in the model have the following physical interpretation:

$$\alpha = \frac{rF_z}{J}, \quad \beta = \frac{1}{J}, \quad \gamma = \frac{F_z}{m}$$

where,

m - mass of the quarter car

F_z - vertical force

r - wheel radius

J - wheel inertia

An additional time delay (τ) is added due to sampling and communication between the different modules of the system.

The longitudinal tire slip (λ) definition will imply that a locked wheel ($\omega = 0$) is described by $\lambda = 1$, while the free motion of the wheel ($\omega r = v$) is described by $\lambda = 0$.

The tire friction force, is determined by $F_z\mu(\lambda, \mu_H, \alpha, F_z, v)$ where $\mu(\lambda, \mu_H, \alpha, F_z)$ is the road-tire friction coefficient. This is a nonlinear function with a typical dependence on the slip shown in Figure 1. The most common tire friction model used in the literature is the "Magic Formula" [1], or Pacejka model. This model uses static maps to describe dependence between slip and friction and it can depend on the vehicles velocity (v). This function depends also on the normal force (F_z), steering angle (α), road surface (having different maximum values μ_H for different road conditions). For ease of writing, the model equations highlight only the dependence on the longitudinal tire slip (λ). In Figure 1 there are shown tire friction curves, generated by the Pacejka model, for four different kind of surfaces.

Notice that this model contains a quite simple description of the slip dynamics for a wheel. It does not capture pitching motion of the car body while braking, suspension dynamics, actuator dynamics, tire dynamics nor camber angle (in the above given model, the tire is consider perpendicular on the road surface). However, it captures the major control challenges of the problem.

1. References

[1] E. Bakker, H. B. Pacejka, and L. Lidner. “A new tire model with application in vehicle dynamics studies.” *SAE, 890087*, 1989.

[2] P. A. Bliman, T. Bonald, and M. Sorine. “Hysteresis Operators and Tire Friction Models: Application to vehicle simulation.” In *Proceedings of ICIAM’95, Hamburg, Germany*, 1995.

[3] C. Canudas de Wit and P. Tsiotras. “Dynamic Tire Friction Models.” In *Proceedings of the IEEE Conference on Decision and Control, Phoenix*, pp. 3746–3751, 1999.

[4] S. Drakunov, Ü. Özgüner, P. Dix, and B. Ashrafi. “ABS control using optimum search via sliding mode.” In *IEEE Transactions on Control System Technology*, vol. 3, pp. 79–85, 1995.

[5] F. Gustafsson. “Slip-based Tire-road Friction estimation.” *Automatica*, **33:6**, pp. 1087– 1099, 1997.

[6] T. Johansen, J. Kalkkuhl, J. Lüdemann, and I. Petersen. “Hybrid Control Strategies in ABS.” *Proceedings of the American Control Conference, Arlington*, 2001.

[7] Y. Liu and J. Sun. “Target slip tracking using gain-scheduling for braking systems.” In *Proceedings of the American Control Conference, Seatle, Washington*, pp. 1178–1182, 1998.

[8] S. Solyom. “Synthesis of a model-based tire slip controller.” Technical Report Licentiate thesis ISRN LUTFD2/TFRT–3228–SE, Department of Automatic Control, Lund Institute of Technology, Sweden, June 2002.

[9] P. Wellstead and N. Pettit. “Analysis and redesign of an antilock brake system controller.” In *IEE Proceedings - Control Theory Appl.*, vol. 144, pp. 413–425, 1997.